Competition for Content Spread over Multiple Social Networks

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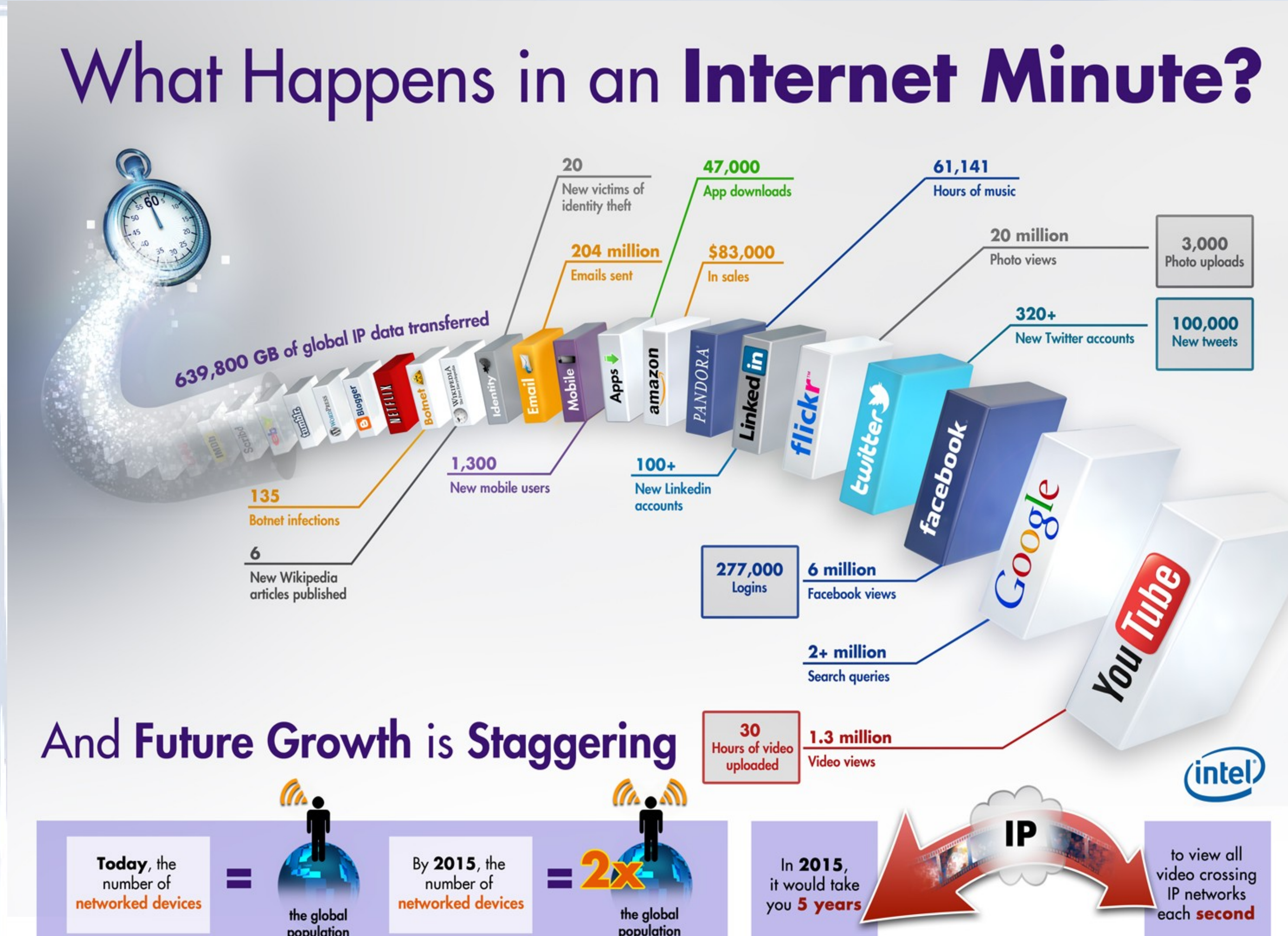
Problem Motivation

Online Social Networks (OSNs)

- Tremendous growth of OSNs
  - **Facebook** (>1 billion), **Google+**(500 million), **Twitter** (300 million), **LinkedIn** (250 million)
  - Increased user presence and activity levels
- Expected to grow further with
  - Smartphones, tablets, wearable computing
  - Advances in mobile Internet
- Timely & targeted delivery of content as opposed to traditional media (newspapers, television, etc.)
- A better platform for brands to advertise their products



Source: dailyinfographic.com

Literature Survey

- Viral marketing for influence/information spread
  - Threshold and Cascade models
  - Most influential nodes (Kempe et al. '03, Leskovec et al. '07)
- Fluid limit based approaches
  - motivated by large userbase of these networks
  - yields differential equations capturing average dynamics
  - equivalence between threshold model and epidemic models (Srinivasan and Kumar '12)
- Epidemic control and games
  - Modeling incentive and strategies (Dayama et al. '12)
  - Modeling competition between content creators (Altman '12)

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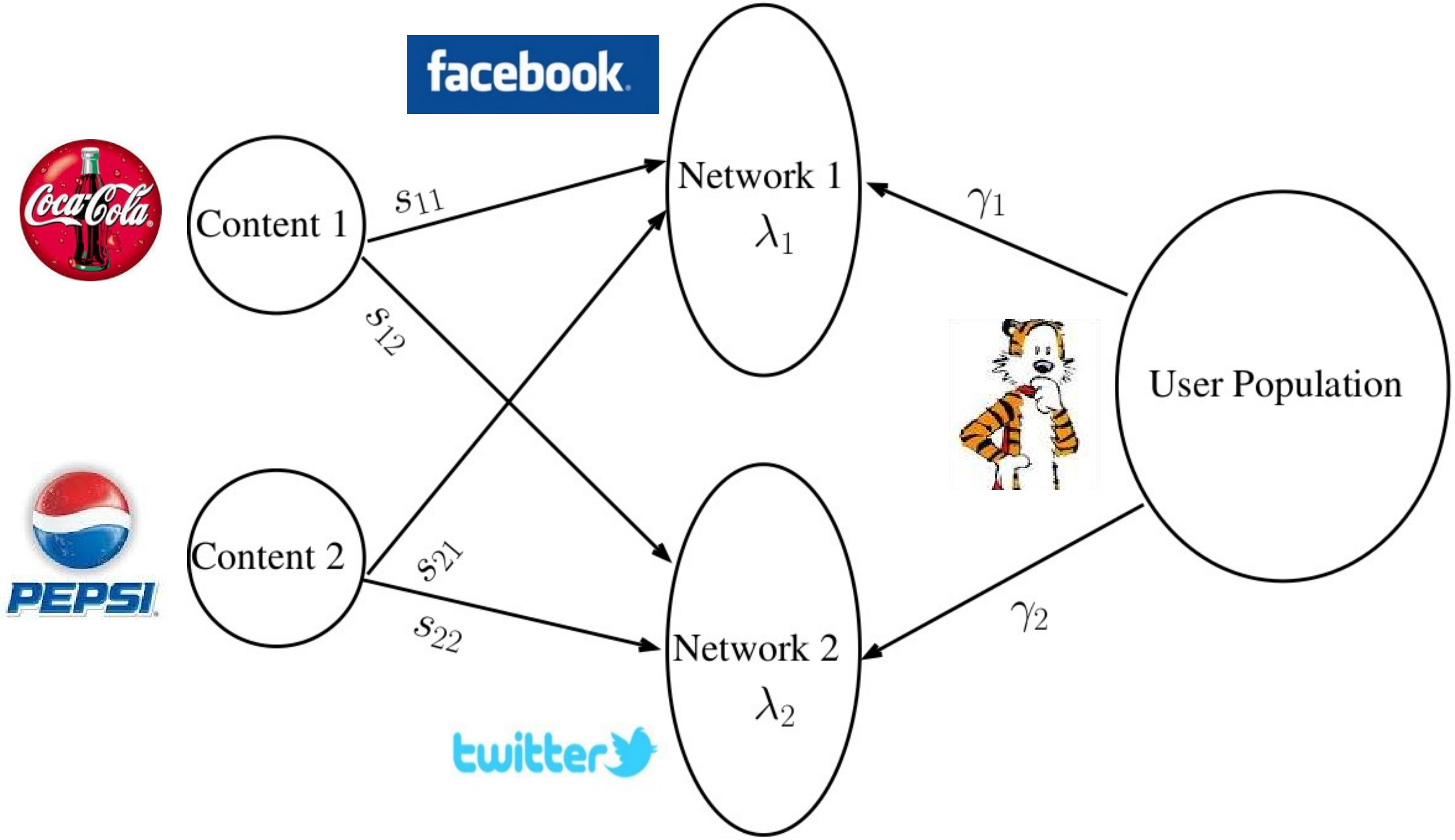
Multiple Social Networks

- Prior literature: <u>the focus is on a single social network</u>
  - Users spend time across several social networks
  - Migration patterns in social media (Shamanth et al. '11)
- Each network has evolved for a different purpose
  - facebook (status updates), twitter (shared interests), LinkedIn (professional contacts), etc.
  - OSNs vary in their **level of popularity** (fraction of user base) and the **level of activity** (content update rate) within the network
- Competing brands must optimally allocate their advertising budget across multiple social networks

System Model

System Model



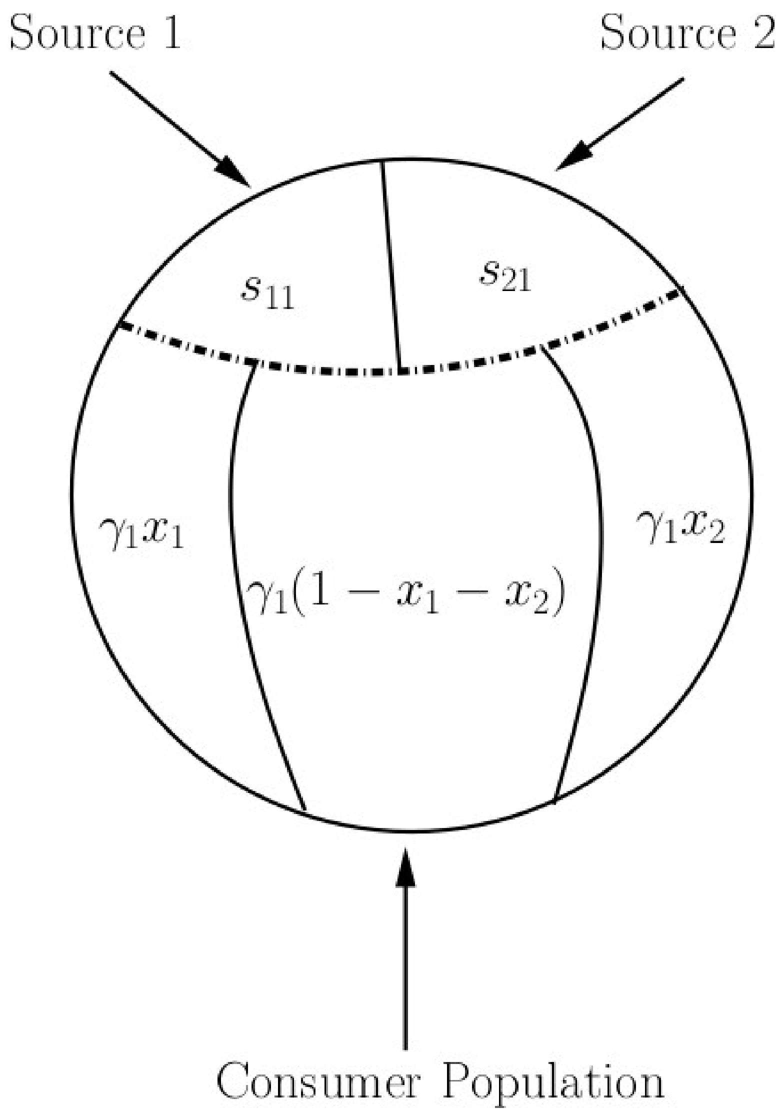
γ - Level of popularity (visit probability per user to a network)

λ - Level of activity (meeting rate within a network)

s_{ij} – 'investment' of source i in network j

- number of accounts/pages maintained, number of initial seed users, etc.

System Model (contd.)



Network 1

Assumption: Contents are mutually exclusive

Content adoption

- x_1 – fraction of users who have adopted content 1
- x_2 – fraction of users who have adopted content 2

$$x_1 + x_2 \leq 1$$

Simplification: We do not model content sharing **between** users in this work (restriction to **publish-subscribe** framework)

Epidemic spread

On visiting network i (occurs w.p. γ_i) users meet content sources in that network (pairwise) at points of a Poisson process of rate λ_i

After suitable normalization, we assume $s_{ij} \leq 1$

Content Dynamics

System evolution (transfer only from source):

$$\dot{x}_1 = (s_{11}\lambda_1\gamma_1 + s_{12}\lambda_2\gamma_2)(1 - x_1 - x_2)$$
$$\dot{x}_2 = (s_{21}\lambda_1\gamma_1 + s_{22}\lambda_2\gamma_2)(1 - x_1 - x_2)$$

Let $a = \lambda_1\gamma_1$, $b = \lambda_2\gamma_2$, and $X_i := x_i(\infty)$. Then we have,

$$X_i = \frac{as_{i1} + bs_{i2}}{as_{i1} + bs_{i2} + as_{j1} + bs_{j2}}$$

Ratio of appropriately weighted sum of the investments

Effectiveness of network j for content spread in this model depends on the product $\lambda_j\gamma_j$

Game Formulation

Game Formulation

- Let the utility function be given by

$$U_i = X_i - \phi_i(s_{i1} + s_{i2})$$

- With $s_{ij} \le 1$ let the total budget for source i be $B_i = s_{i1} + s_{i2}$
- Without loss of generality, assume that $\lambda_1\gamma_1 > \lambda_2\gamma_2$
- Then we can show that in the optimal budget allocation, we have

$$s_{i1} = \min(B_1, 1) \text{ and } s_{i2} = \min((B_1 - 1)^+, 1)$$

<u>Utility function in terms of the budget</u>

$$U_i = \begin{cases} \frac{aB_i}{aB_i + z_j} - \phi_i B_i, & B_i \le 1 \\ \frac{a-b+bB_i}{a-b+bB_i+z_j} - \phi_i B_i, & 1 \le B_1 \le 2 \end{cases}$$

Best Response Characterization

Let

$$f_1 = \frac{1}{a}\left(\sqrt{\frac{z_2 a}{\phi_1}} - z_2\right)$$

"utility" of investing only in network 1

$$g_1 = \frac{1}{b}\left(\sqrt{\frac{z_2 b}{\phi_1}} - a + b - z_2\right)$$

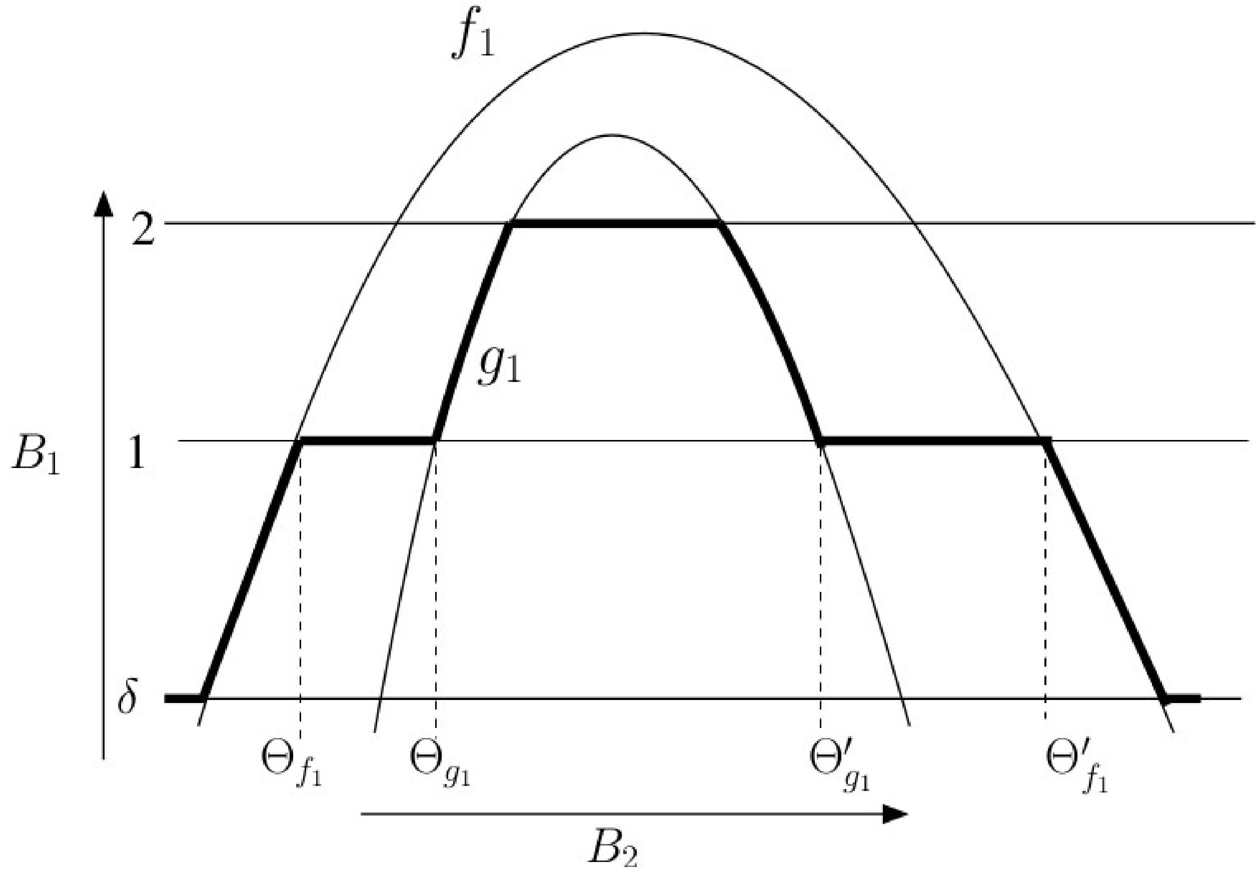
"utility" of saturating network 1, and investing in network 2

where $z_2 = a\min(B_2, 1) + b\min((B_2 - 1)^+, 1)$

Best response function for player 1 (and similarly for player 2)

$$BR_1(B_2) = \begin{cases} \max(f_1, \delta), & 0 < B_2 < \Theta_{f_1} \\ 1, & \Theta_{f_1} < B_2 < \Theta_{g_1} \\ \min(g_1, 2), & \Theta_{g_1} < B_2 < \Theta'_{f_1} \\ 1, & \Theta'_{g_1} < B_2 < \Theta'_{f_1} \\ \max(f_1, \delta), & B_2 > \Theta'_{f_1} \end{cases}$$

Structure of Best Response Curve



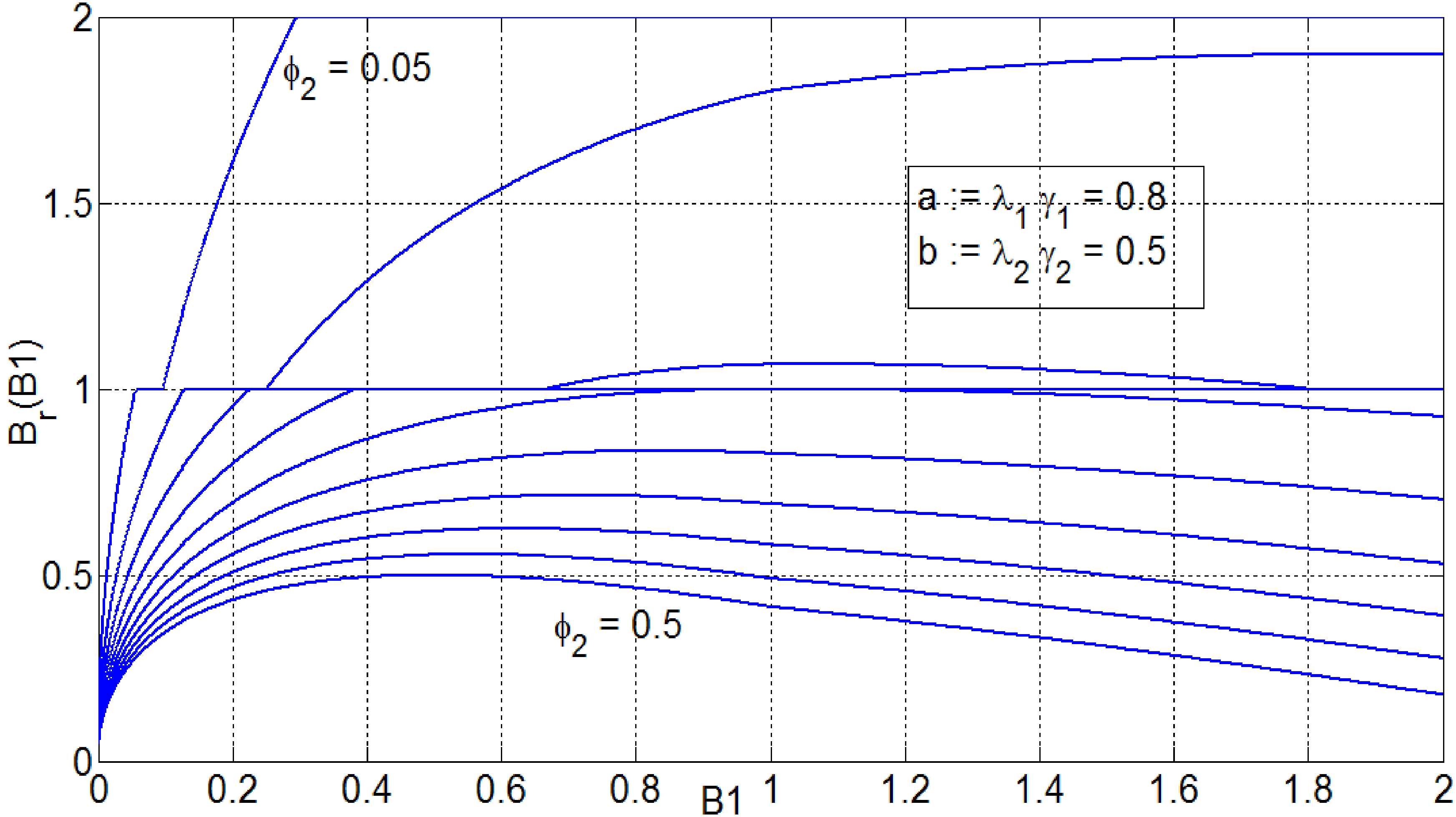
- Players need to decide among five choices of budget range for best response:
  - **δ** , **(δ, 1)** , **1**, **(1, 2)**, **2**

- Observe that player 1, **after saturating Network 1, does not *immediately* start investing in Network 2** for increasing values of opponent budget **(hysteresis)**.

- **Similarly**, when responding to much higher opponent budgets, **after withdrawing from Network 2**, he does not *immediately* start reducing investment in Network 1.

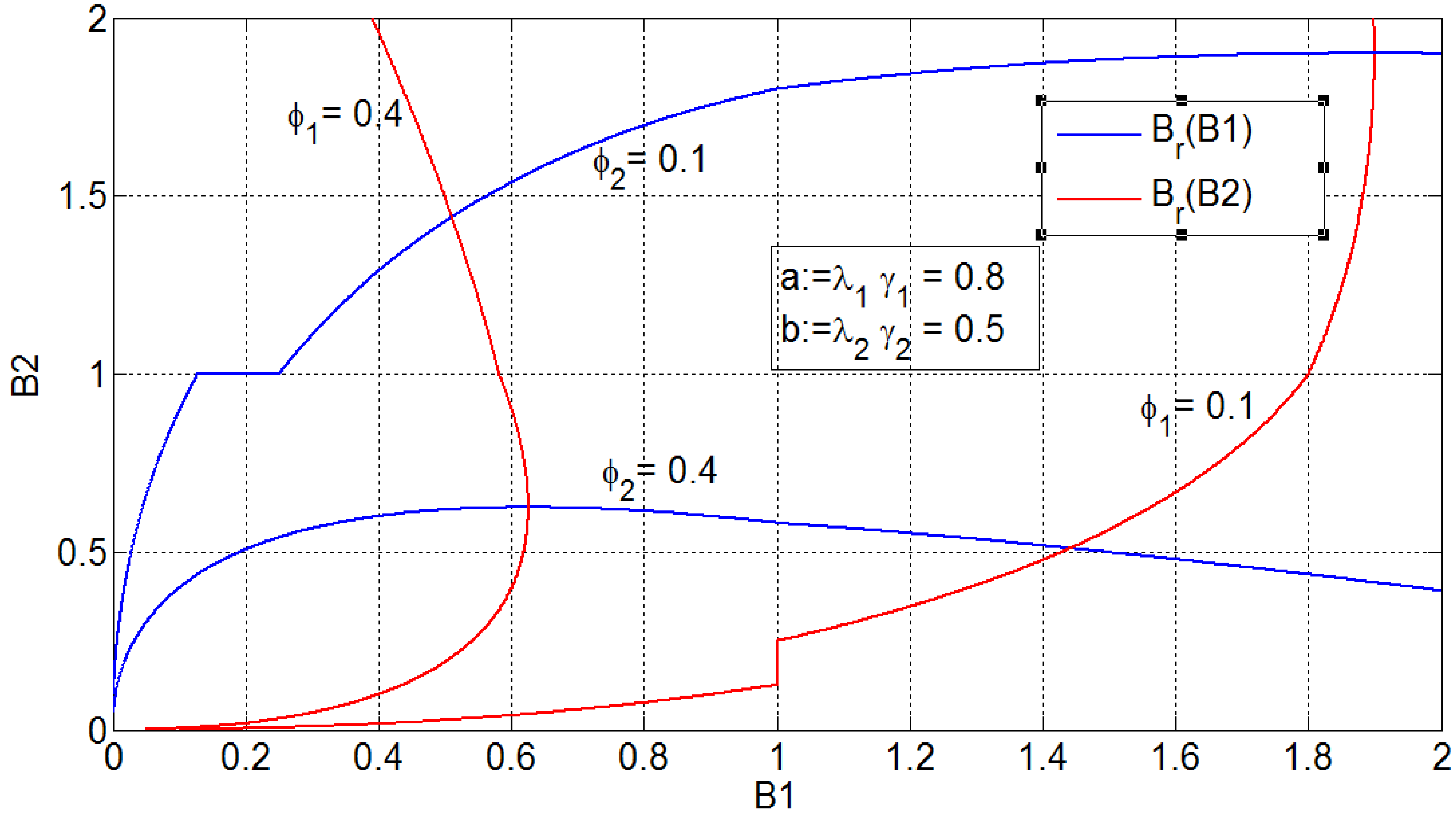
Numerical Results

Best response of Player 2



Observe that as the cost term increases, the player becomes **less aggressive.**

Nash Equilibrium



- The Nash equilibrium can be obtained as the intersection of the best response curves.
- **Symmetric costs:** We get a symmetric Nash equilibrium.
- **Non-symmetric costs:** Player with a higher cost term, operates at a lower budget.

Remarks on the Utility Function

Recall the utility function given by:

$$U_i = X_i - \phi_i(s_{i1} + s_{i2})$$

with $X_i = \dfrac{as_{i1} + bs_{i2}}{as_{i1} + bs_{i2} + as_{j1} + bs_{j2}}$

- When $\lambda_1\gamma_1 = \lambda_2\gamma_2$, we have a **Cournot duopoly game** with budgets as the strategies and an ***iso-elastic* inverse demand function** $P(B_1, B_2) = \frac{1}{B_1+B_2}$

- Similarly, when the cost is of the form $\phi_1 s_{i1} + \phi_2 s_{i2}$, with $\phi_j = C\lambda_j\gamma_j$, we again get a Cournot duopoly game with iso-elastic demand function, but with the strategies $Q_i = \lambda_1\gamma_1 s_{i1} + \lambda_2\gamma_2 s_{i2}$

Cournot duopoly:
Price (Demand) = Inverse function of total production (usually linear)
Utility = My production * (Price - Cost per unit)

Conclusion

- We have studied content competition across multiple social networks.
- Established the hysteresis structure of best response curves
- Obtained the Nash equilibrium numerically as intersection of the best response curves
- Provided interesting insights into the utility function, which is a generalization of the Cournot duopoly model

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Thank you

Backup slides

Content Dynamics (with sharing)

System evolution (transfer from source + sharing among users):

$$\dot{x}_1 = (s_{11} + \alpha_{11}\gamma_1 x_1)\lambda_1\gamma_1(1 - x_1 - x_2) + (s_{12} + \alpha_{12}\gamma_2 x_1)\lambda_2\gamma_2(1 - x_1 - x_2)$$

$$\dot{x}_2 = (s_{21} + \alpha_{21}\gamma_1 x_2)\lambda_1\gamma_1(1 - x_1 - x_2) + (s_{22} + \alpha_{22}\gamma_2 x_2)\lambda_2\gamma_2(1 - x_1 - x_2)$$

System evolution (transfer only from source):

$$\dot{x}_1 = (s_{11}\lambda_1\gamma_1 + s_{12}\lambda_2\gamma_2)(1 - x_1 - x_2)$$

$$\dot{x}_2 = (s_{21}\lambda_1\gamma_1 + s_{22}\lambda_2\gamma_2)(1 - x_1 - x_2)$$